$$\therefore \rho = r\sqrt{\left(\frac{l\,\frac{n-n_r}{n}}{l\,\frac{1}{2}}\right)} = r\sqrt{\left(\frac{l\,\frac{n}{n-n_r}}{l\,2}\right)}. \tag{10}$$

This agrees with Herschel's formulæ which Prof. Chase employs.

I abstain from developing the more accurate formulæ for determining these constants from the sum of the squares of the distances, or from the sum of the distances (the string), also in the case of stray shots, and give in conclusion the most probable shots of the marksmen A and B. They are,

For marksman A, $\varepsilon_a = \dfrac{5}{\sqrt{(2l\,\frac{100}{64})}} = 5.292.$

For marksman B, $\varepsilon_b = \dfrac{10}{3\sqrt{(2l\,\frac{100}{36})}} = 4.664.$

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PROBLEM 436.—"Integrate the equation

$$x^m y^n (aydx + bxdy) = x^{m'} y^{n'} (a'ydx + b'xdy)."$$

SOLUTION BY PROF. W. W. BEMAN.—Evidently $x^{-m-1}y^{-n-1}$ is an integrating factor of the first member, whence an integral of $x^m y^n(aydx+bxdy) = 0$, is $\log (x^a y^b) = c$, or $x^a y^b = C$.

The general form of integrating factor of the first member is, then,

$$x^{-m-1}y^{-n-1}\,\varphi(x^a y^b).$$

In the same way we may obtain a general integrating factor for the second member, $x^{-m'-1}y^{-n'-1}\,\psi(x^{a'} y^{b'})$.

That these two may be equal, we must have

$$x^{m'+1}y^{n'+1}\,\varphi(x^a y^b) = x^{m+1}\,y^{n+1}\,\psi(x^{a'} y^{b'}).$$

Let $\varphi(x^a y^b) = (x^a y^b)^r$, $\psi(x^{a'} y^{b'}) = (x^{a'} y^{b'})^s$, $r$ and $s$ being indeterminate. Then

$$x^{m'+1}\,y^{n'+1}\,(x^a y^b)^r = x^{m+1}\,y^{n+1}\,(x^{a'} y^{b'})^s.$$

$$\therefore\; m'+ar = m+a's, \quad n'+br = n+b's;$$

$$r = \frac{a'(n-n')-b'(m-m')}{a'b-ab'}, \quad s = \frac{a(n-n')-b(m-m')}{a'b-ab'}.$$

The integrating factor becomes

$$x^{ar-m-1}\,y^{br-n-1} = x^{a's-m'-1}\,y^{b's-n-1};$$

$$\therefore\; x^{ar-1}\,y^{br-1}(aydx+bxdy) = x^{a's-1}\,y^{b's-1}(a'ydx+b'xdy).$$

Integrating, $\dfrac{1}{r}x^{ar}\,y^{br} = \dfrac{1}{s}x^{a's}\,y^{b's} + C.$

When $a'b = ab'$ the equation is immediately integrable.

This solution is based upon that of a special form of the equation given, found in Hoüel's Calcul Infinitesimal.